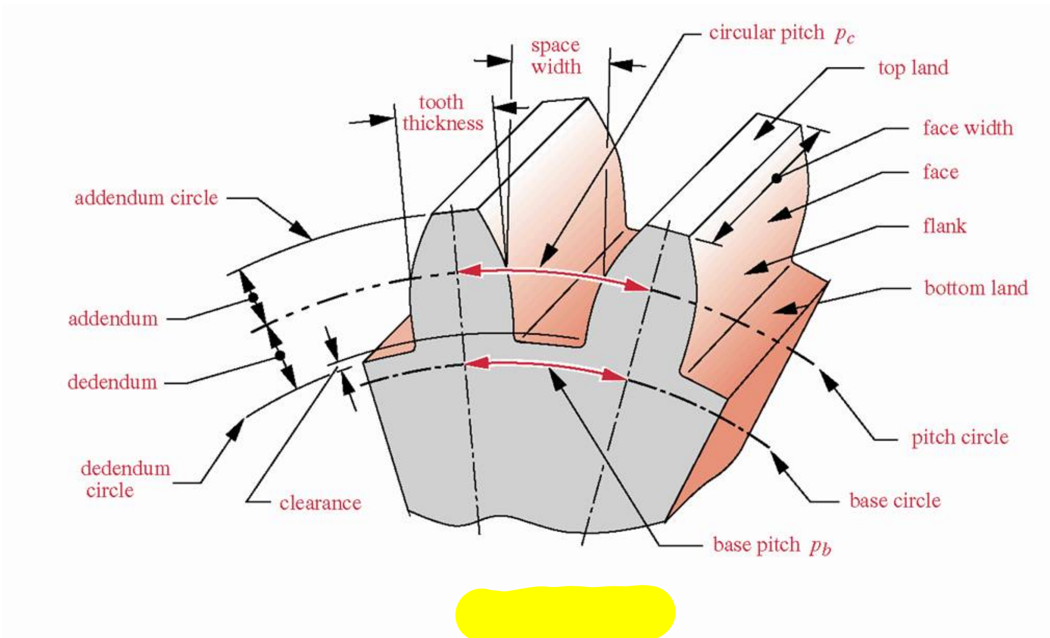
ME 314 - Engineering Design : Mechanical Components
Lecture 23

Note Title

Example 1- Determining Gear Tooth and Gear Mesh Parameters

A spur pinion transmits 15 hp at 1200 rpm. It has a pitch of 6 teeth per in, 22 full-depth teeth, and a 20° pressure angle. The gear has 60 teeth. Find the gear ratio, circular pitch, base pitch, pitch diameters, pitch radii, base radii, center distance, addendum, dedendum, whole depth, clearance, outside diameters, and contact ratio of the gearset. If the center distance is increased by 3% what is the new pressure angle?



Gear Tooth Nomenclature.

Solution :

Gear ratio - Eq. 12.5b :

$$m_G = \frac{N_g}{N_p} \qquad (a)$$

Circular pitch - Eq. 12.4b :

$$p_c = \frac{\pi}{P_d}$$

Base pitch - Eq. 12.3b

$$p_b = p_c \cos\phi$$

Pitch diameters and **radii** of pinion and gear – Eq. 12.4a

$$d_p = \frac{N_p}{P_d} \quad , \quad r_p = \frac{d_p}{2}$$

$$d_g = \frac{N_g}{P_d} \quad , \quad r_g = \frac{d_g}{2}$$

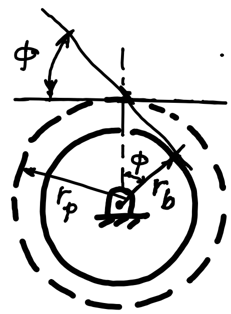
Base diameters and **radii** of pinion and gear – Not in text:

$$(r_b)_p = r_p \cos\phi$$

$$(r_b)_g = r_g \cos\phi$$

$$(d_b)_p = 2(r_b)_p$$

$$(d_b)_g = 2(r_b)_g$$



Center distance is the sum of pitch radii

$$C = r_p + r_g$$

Addendum and dedendum are found from Eqs. in Table 12-1.

Parameter	Coarse Pitch (p_d < 20)	Fine Pitch ($p_d \geq 20$)
Pressure angle ϕ	20° or 25°	20°
Addendum a	1.000 / p_d	1.000 / p_d
Dedendum b	1.250 / p_d	1.250 / p_d
Working depth	2.000 / p_d	2.000 / p_d
Whole depth	2.250 / p_d	2.200 / p_d + 0.002 in
Circular tooth thickness	1.571 / p_d	1.571 / p_d
Fillet radius—basic rack	0.300 / p_d	not standardized
minimum basic clearance	0.250 / p_d	0.200 / p_d + 0.002 in
minimum width of top land	0.250 / p_d	not standardized
Clearance (shaved or ground teeth)	0.350 / p_d	0.350 / p_d + 0.002 in

AGMA Full-Depth Gear Tooth Specifications.

$$a = \frac{1.0}{P_d} = \frac{1.0}{6} = 0.167 \text{ in}, \quad b = \frac{1.25}{P_d} = \frac{1.25}{6} = 0.208 \text{ in}$$

Whole depth $h_t = a + b = 0.167 + 0.208 = 0.375$ in

Clearance, $c = b - a = 0.208 - 0.167 = 0.042$ in

Outside diameter = pitch diameter + 2 (addendum)

$$(D_o)_p = d_p + 2a = 3.67 + 2(0.167) = 4.00 \text{ in}$$

$$(D_o)_g = d_g + 2a = 10.0 + 2(0.167) = 10.33 \text{ in}$$

Contact ratio, m_p is found from Eqs. 12.2 and 12.7:

First, length of action is

$$Z = \sqrt{(r_p + a_p)^2 - (r_p \cos\phi)^2} + \sqrt{(r_g + a_g)^2 - (r_g \cos\phi)^2} - C \sin 20^\circ$$

$$= \sqrt{(r_o)_p^2 - (r_b)_p^2} + \sqrt{(r_o)_g^2 - (r_b)_g^2} - C \sin 20^\circ$$

$$= \sqrt{\left(\frac{4.00}{2}\right)^2 - (1.72)^2} + \sqrt{\left(\frac{10.33}{2}\right)^2 - (4.70)^2} - 6.83 \sin 20^\circ$$

$$= 0.826 \text{ in}$$

$$m_p = \frac{Z}{P_b} = \frac{0.826}{0.492} = 1.75$$

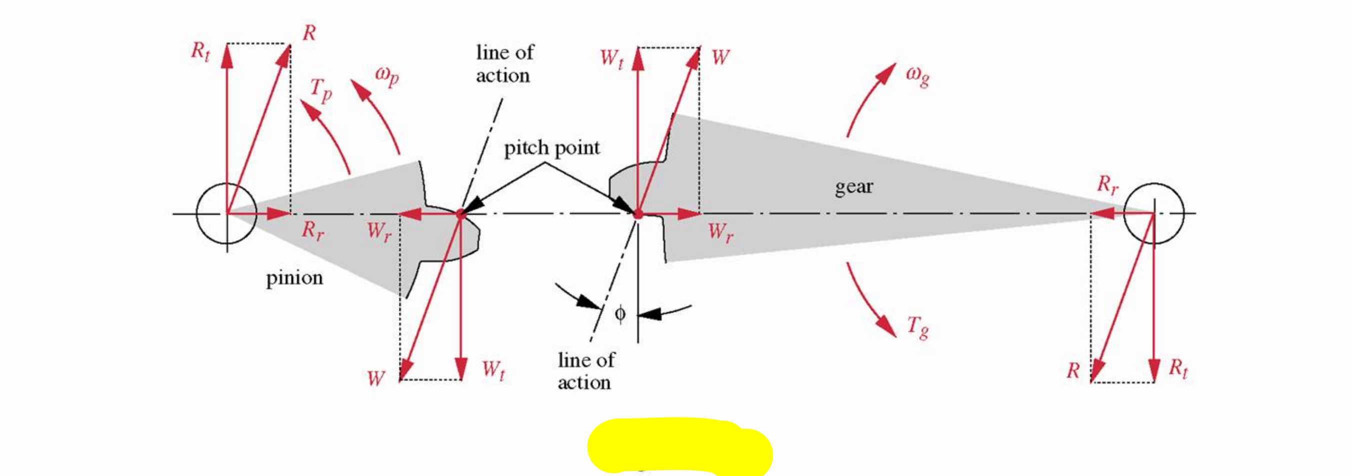
Note that since $1 < m_p < 2$, the load is applied at the Highest Point of Single-Tooth Contact (HPSTC).

If the center distance is increased due to assembly error or other reasons, the pitch radii will change by the same percentage but the gears' base radii will remain the same (recall the definitions of pitch and base circles): $(r_b)_p = (1.03\, r_p) \cos\phi_{new}$

$$1.72 = (1.03)(1.83) \cos\phi_{new} \Rightarrow \phi_{new} = 24.14^\circ$$

12.7 Loading on Spur Gears

A pair of gear teeth in contact at the pitch point, but shown separated for clarity, are shown in Fig. 12-20.



Forces on the Pinion and Gear in a Gearset (Gears Separated for Illustration—Pitch Points are Actually in Contact).

The torque T_p is transmitted by the pinion to the gear. Neglecting friction at the point of contact, i.e., the pitch point, the force applied by the pinion on the gear is W in the direction of the line of action (i.e., normal to the common tangent). W can be resolved into a radial component, W_r, and a tangential component, W_t. We have

$$W_t = \frac{T_p}{r_p} = \frac{T_p}{d_p/2} = \frac{2P_d T_p}{N_p} \qquad (12.13a)$$

where r_p is the pitch radius,
d_p the pitch diameter,
N_p the number of teeth
P_d the diametral pitch of the pinion

We have

$$W_r = W_t \tan\phi \qquad (12.13b)$$

and

$$W = \frac{W_t}{\cos\phi} \qquad (12.13c)$$

Note that (12.13a) could have been written for the gear. The reaction force R is equal and opposite to W.

Example 2- Load Analysis of a Spur Gearset

A spur pinion transmits 15 hp at 1200 rpm. It has a pitch of 6 teeth per in, 22 full-depth teeth, and a 20° pressure angle. The gear has 60 teeth. Determine the torque and the transmitted load on the gear teeth. What are the mean and alternating components of the transmitted load?

Solution:

1. The pitch diameter for pinion is

$$d_p = \frac{N}{P_d} = \frac{22}{6} = 3.67 \text{ in}$$

2. The torque on the pinion shaft is found from Eq. 10.1:

$$T_p = \frac{P}{\omega_p} = \frac{15 \text{ hp} \left(6600 \ \frac{\text{lb-in}}{\text{s}}/\text{hp}\right)}{1200 \text{ rpm} \left(\frac{2\pi}{60} \ \frac{\text{rad/s}}{\text{rpm}}\right)} = 787.8 \text{ lb-in}$$

3. The output torque is

$$T_g = m_G T_p = \frac{N_g}{N_p} T_p = \frac{60}{22}(787.8) = 2148 \text{ lb-in}$$

4. The transmitted load is found from the torque & radius:

$$W_t = \frac{T_p}{d_p/2} = \frac{787.8}{3.67/2} = 429.7 \text{ lb}$$

5. The radial component of the load is

$$W_r = W_t \tan\phi = (429.7)\tan 20^\circ = 156.4 \text{ lb}$$

6. The total load is

$$W = \frac{W_t}{\cos\phi} = \frac{429.7}{\cos 20^\circ} = 457.3 \text{ lb}$$

7. The repeated load on pinion and gear are

$$(W_t)_{alternating} = \frac{429.7}{2} = 214.8 \text{ lb}, \quad (W_t)_{mean} = \frac{429.7}{2} = 214.8 \text{ lb}$$

12.8 Stresses in Spur Gears

Gear teeth fail either by fatigue fracture due to fluctuating bending stresses at the root of the tooth, or by surface fatigue (pitting) of the tooth surfaces. We need to check for both failure modes when we design gears. To check for fatigue fracture, we employ the modified-Goodman line. Since most gears are made of ferrous materials and have an endurance limit for bending, we can design them for infinite life as far as bending is concerned. However, materials do not have an endurance limit for repeated contact stresses. Hence, we cannot design gears for infinite life against surface fatigue failure. Pitting is the most common mode of failure. Abrasive or adhesive wear in the form of scuffing or scoring may also occur.

Bending Stresses

Modeling the gear tooth as a cantilever beam with its critical section at the root, Lewis (1892) started with the equation for bending stress and derived what is now known as the Lewis equation:

$$\sigma_b = \frac{W_t P_d}{F Y} \qquad (12.14)$$

where

W_t = Tangential force on the tooth

P_d = Diametral pitch

F = Face width

Y = A dimensionless geometry factor
Now called the Lewis Form Factor

Stress concentration was not yet discovered in 1892. Lewis's form factor Y has been replaced by a "geometry factor" J, which includes the effect of stress concentration at the root fillet. Also AGMA has introduced additional factors to account for various failure mechanisms discussed above. The AGMA's bending stress equation has the form:

$$\sigma_b = \frac{W_t P_d}{F J} \frac{K_a K_m}{K_v} K_s K_B K_I \qquad (12.15)\text{US}$$

Or

$$\sigma_b = \frac{W_t}{F m J} \frac{K_a K_m}{K_v} K_s K_B K_I \qquad (12.15)\text{SI}$$

where the K factors are modifiers to account for various conditions,and m is the module. The geometry factor J can be calculated from an algorithm given in AGMA Standard 908-B89. Using this standard, J values are tabulated for various tooth conditions and pressure angles.

Gear teeth	Pinion teeth 12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	U	U										
21	U	U	U	U	U	U	0.24	0.24								
26	U	U	U	U	U	U	0.24	0.25	0.25	0.25						
35	U	U	U	U	U	U	0.24	0.26	0.25	0.26	0.26	0.26				
55	U	U	U	U	U	U	0.24	0.28	0.25	0.28	0.26	0.28	0.28	0.28		
135	U	U	U	U	U	U	0.24	0.29	0.25	0.29	0.26	0.29	0.28	0.29	0.29	0.29

Under cutting Occur

AGMA Bending Geometry Factor J for 20°, Full-Depth Teeth with Tip Loading.

Equal addenda on both pinion and gear

(Tip of gear tooth and root-flank of pinion interfer)

Gear teeth	Pinion teeth 12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	U	U										
21	U	U	U	U	U	U	0.33	0.33								
26	U	U	U	U	U	U	0.33	0.35	0.35	0.35						
35	U	U	U	U	U	U	0.34	0.37	0.36	0.38	0.39	0.39				
55	U	U	U	U	U	U	0.34	0.40	0.37	0.41	0.40	0.42	0.43	0.43		
135	U	U	U	U	U	U	0.35	0.43	0.38	0.44	0.41	0.45	0.45	0.47	0.49	0.49

AGMA Bending Geometry Factor J for 20°, Full-Depth Teeth with HPSTC Loading.

Highest Point of Single-Tooth Contact

For precision gears, load sharing between the teeth occurs and HPSTC tables can be used. If not, then it is likely that only one pair of teeth will take all the load at the tip.

Gear teeth	Pinion teeth 12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	0.27	0.19										
21	U	U	U	U	0.27	0.21	0.27	0.21								
26	U	U	U	U	0.27	0.22	0.27	0.22	0.28	0.22						
35	U	U	U	U	0.27	0.24	0.27	0.24	0.28	0.24	0.28	0.24				
55	U	U	U	U	0.27	0.26	0.27	0.26	0.28	0.26	0.28	0.26	0.29	0.26		
135	U	U	U	U	0.27	0.28	0.27	0.28	0.28	0.28	0.28	0.28	0.29	0.28	0.30	0.28

AGMA Bending Geometry Factor J for 20°, 25%-Long-Addendum Teeth with Tip Loading.

Gear addendum is 25% shorter while pinion addendum is 25% longer

Gear teeth	Pinion teeth 12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	0.36	0.24										
21	U	U	U	U	0.37	0.26	0.39	0.27								
26	U	U	U	U	0.37	0.29	0.39	0.29	0.41	0.30						
35	U	U	U	U	0.37	0.32	0.40	0.32	0.41	0.33	0.43	0.34				
55	U	U	U	U	0.38	0.35	0.40	0.36	0.42	0.36	0.44	0.37	0.47	0.39		
135	U	U	U	U	0.39	0.39	0.41	0.40	0.43	0.41	0.45	0.42	0.48	0.44	0.51	0.46

AGMA Bending Geometry Factor J for 20°, 25%-Long-Addendum Teeth with HPSTC Loading.

Dynamic Factor K_V

The ideal of smooth, constant-velocity-ratio torque transmission (fundamental law of gearing) is closely achieved for precision gears. However, when manufacturing tolerances are large (low-precisian gears), there will be internally generated vibration loads from tooth-tooth impacts induced by non-conjugate meshing of gear teeth. The dynamic factor, K_V which is introduced by AGMA to account for this load is given by the empirical equations:

$$K_V = \left(\frac{A}{A+\sqrt{V_t}} \right)^B \qquad \text{(12-16) US}$$

or

$$K_v = \left(\frac{A}{A+\sqrt{200V_t}}\right)^B \qquad (12.16)\,SI$$

where V_t is the pitch-line velocity of the gear mesh in ft/min (U.S.) or m/s (SI).

$$A = 50 + 56(1-B) \qquad (12.17a)$$

$$B = \frac{(12-Q_v)^{2/3}}{4} \quad \text{for } 6 < Q_v < 11 \qquad (12.17b)$$

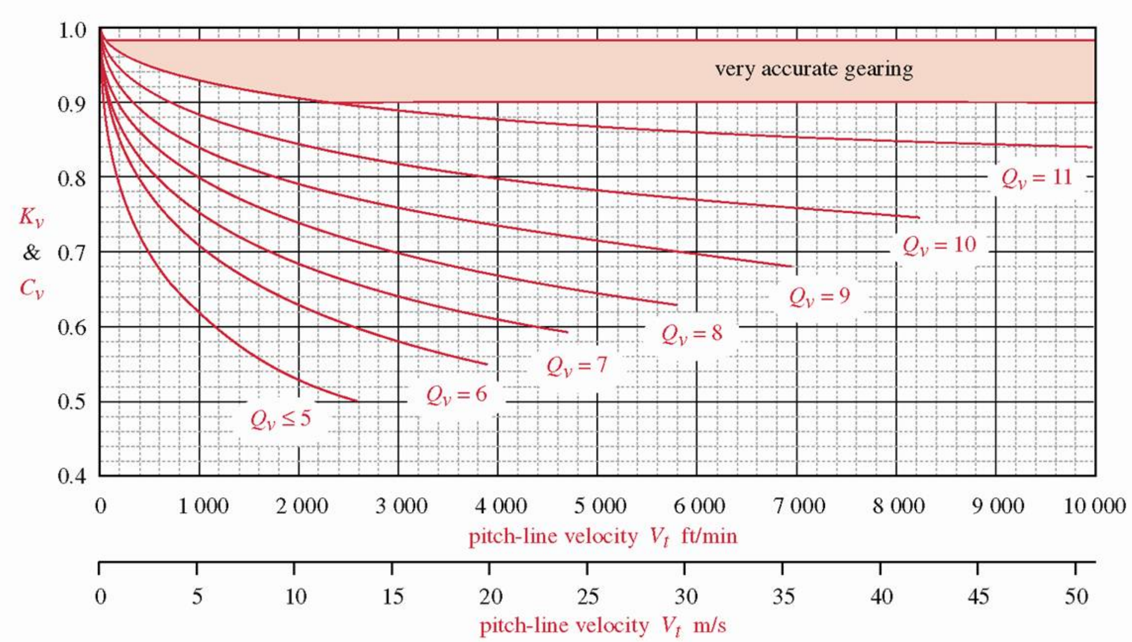
Q_v = The quality index of the lower-quality gear in the mesh

For gears with $Q_v \leqslant 5$, K_v is given by

$$K_v = \frac{50}{50+\sqrt{V_t}} \qquad (12.19)\,US$$

$$K_v = \frac{50}{50+\sqrt{200V_t}} \qquad (11.19)\,SI$$

These are valid for $V_t \le 2500$ ft/min (13 m/s). Above this velocity, gears of higher quality should be used. The above equations are plotted in Fig. 12-22.



AGMA Dynamic Factors K_v and C_v.

Remarks on the gear quality (see Section 11.6 for details).

There are two methods of manufacturing gears: Forming and machining. Forming refers to the casting, molding, drawing, or extrusion of tooth forms. Machining is divided into roughing (milling, rack generation, shaping, and hobbing) and finishing (shaving, grinding, burnishing, lapping and honing). Roughing and finishing are used when high precision and quiet running are required.

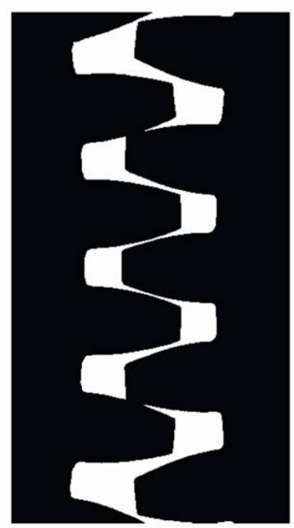
The AGMA Standard 2000- A88 defines dimensional tolerances for gear teeth and a quality index $3 \leq Q_V \leq 16$. For example, AGMA recommends a $3 \leq Q_V \leq 5$ for a cement mixer drum drive while for a gyroscope the recommendation is $12 \leq Q_V \leq 14$ (see Table 12-6, for more examples).

Another way to select a suitable Q_V is based on the linear velocity of the gear teeth at the pitch point, V_t . Table 11-7 shows recommended values.

Note that helical gears are preferred for V_t > 10,000 ft/min.

Pitch Velocity	Q_v
0–800 fpm	6–8
800–2000 fpm	8–10
2000–4000 fpm	10–12
Over 4000 fpm	12–14

Gear quality can have a significant effect on the load sharing between teeth. If the tooth spacings are not accurate and uniform, the teeth in the mesh will not all be in simultaneous contact. This can eliminate the advantage gained by a large contact ratio. Fig. 12-19 shows two gears with large contact ratio but low accuracy. Only one pair are in contact and taking load and despite a contact ratio of 4-5, the actual contact ratio is only 1.



Application Factor K_a

The load analysis of Section 12.7 was based on the assumption that the torque T_p delivered by the pinion and the resulting tangential force W_t at the pitch point were not varying with time (M_a and T_a are due to the teeth coming into and out of mesh under a uniform load). If either driving or driven machine has time-varying torque or forces, the loading felt by the gear teeth will increase.

The application factor K_a is introduced to correct for increase in stress due to variable torques. Table 12-7 shows values of K_a recommended by AGMA.

	Driven Machine (Load)		
Driving Machine (source)	Uniform	Moderate Shock	Heavy Shock
Uniform (Electric motor, turbine)	1.00	1.25	1.75 or higher
Light Shock (Multicylinder engine)	1.25	1.50	2.00 or higher
Medium Shock (Single-cylinder engine)	1.50	1.75	2.25 or higher

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Application Factors *Ka*.